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Oscillatory behaviour in buoyant thermocapillary convection of fluid layers with a free surface

P. WANG and R. KAHAWITA†

Department of Civil Engineering, Ecole Polytechnique de Montréal, Montréal, Québec, Canada H3C 3A7

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Abstract—Oscillatory behaviour in thermocapillary convection with buoyancy forces has been studied numerically for superposed immiscible liquid layers with a free surface, in which the lower layer consists of low-Prandtl-number fluid. Numerical solutions to the complete two dimensional Navier–Stokes and energy equations have been obtained using the spline integration method. Attention has been focused on flow instabilities of an oscillatory nature which appear to be induced by the buoyancy forces. An attempt to understand the origin of these instabilities and indications on how to reduce or even avoid them is made. The numerical results demonstrate that oscillatory flow in a single layer of low-Prandtl number fluid may transform to a steady state after encapsulation with a fluid of higher Prandtl number, even in the absence of Marangoni forces, except when the buoyancy and the viscous forces in the upper layer are very small when compared with the lower one. The numerical experiments also demonstrate that the addition of the combined Marangoni forces to the gravitational convection plays an important role in suppressing oscillatory flow. © 1997 Elsevier Science Ltd.

INTRODUCTION

Combined buoyancy and thermocapillary convection in differential cavities for a single fluid layer with a free surface has been extensively investigated analytically, numerically and experimentally [1–9] and still remains a current topic of research interest especially in superposed immiscible liquid layers due to its importance in many natural and industrial processes. Some industrial applications that involve thermocapillary forces are surface melting and alloying techniques using high power lasers and processing of ceramics and semiconductors that frequently involve a molten and a gaseous phase. One such important application that has been commercially introduced, is the elimination of evaporation of volatile components and a reduction in thermal convection of a liquid melt by encapsulation with a protective molten material. This has resulted in a significant improvement in the quality of the final product used for the manufacture of semiconductors.

Instabilities that result in oscillatory flow, due to the buoyancy and thermocapillary forces will generally cause a temperature oscillation in the field. Such an oscillation is highly undesirable for many technical processes, e.g. for crystal growth since they are known to contribute to the inhomogeneity of the resulting crystal. Ben Hadid and Roux [6] indicated that the basic mechanisms giving rise to these instabilities are still not well understood. Several experimental investigations cited by these authors have shown that temperature fluctuations, caused by an unsteady buoyancy-driven convective flow, occur when the thermal gradient exceeds a certain critical value.

Chun *et al.* [1] indicated that the origin of the instability in Marangoni convection in a float zone is caused by the growth of some temperature disturbances that originate at the free surface. A temperature perturbation here leads to a corresponding disturbance in the surface tension gradient, resulting in perturbation of the velocity field. Using a linear stability analysis for thermocapillary liquid layers subjected to a horizontal temperature gradient, Smith and Davis [2] found that the mechanism of instability is associated with a balance between heat conduction and heat convection in the layer. The velocity field is important only in so much as it transports heat in the system. Any mechanical aspects of the velocity field, such as viscous dissipation, are not important in terms of the fundamental mechanism of the instabilities. Parmentier *et al.* [9] classified three types of instability according to their behaviour which depended on the values taken by the Prandtl numbers.

For two immiscible liquids superposed in a rectangular cavity with differentially heated end walls, convection is initiated due to the horizontal temperature gradient which gives rise to density differences. However, the effect of surface tension forces at the interface and at the free surface also plays an important role in the convective behaviour, since it influences the mechanical coupling between the two

†Author to whom correspondence should be addressed.

NOMENCLATURE

B	aspect ratio of cavity, L'/H'
g	gravitational acceleration
H'	height of fluid layer
k_i	fluid thermal conductivity
L'	length of cavity
Ma_i	Marangoni number, equation (9)
Nu	Nusselt number
Pr_i	Prandtl number, $(\nu/\alpha)_i$
Ra_i	Rayleigh number, $g\beta_i H'^3(T'_h - T'_c)/\nu_i\alpha_i$
Re_i	Reynolds number, $\gamma_i(T'_h - T'_c)L'/\mu_i\nu_i$
t'	time
t	dimensionless time, $t'\alpha_1/H'^2$
T'	temperature
T'_h	temperature of vertical hot wall
T'_c	temperature of vertical cold wall
T	dimensionless temperature
x, y	dimensionless Cartesian coordinates
u, v	dimensionless velocity components: $u_i = u'_i H'/\alpha_1$, $v_i = v' H'/\alpha_1$
u', v'	dimensional velocity components.

Greek symbols

α_i	thermal diffusivity
β_i	coefficient of thermal expansion
γ_i	$-\partial\sigma_i/\partial T$
μ_i	dynamic viscosity
ν_i	kinematic viscosity
σ_i	surface tension
Ψ	dimensionless stream function
Ω	dimensionless vorticity.

Superscript

$\bar{}$	relative quantities (layer 2 to layer 1).

layers. This has been explained in some detail by Wang and Kahawita [10, 11].

To provide an evaluation of the fundamental mechanism of natural (that is, gravity driven) and thermocapillary (Marangoni) convection, experimental and theoretical investigations in idealized geometries (rectangular cavities) has been reported [12–17] with differentially heated side walls.

In addition, Bénard–Rayleigh convection where heating is from below has been studied quite extensively [18–22]. For example, Renardy and Joseph [18], Renardy and Renardy [19] and Renardy [20] have conducted fairly extensive analytical studies on the stability of the two-layer Bénard system using perturbation theory. Their findings indicate that the onset of instability could be oscillatory. A theoretical and experimental study of two-layer convection in fluids heated from below has been reported by Rasenat *et al.* [21]. Their linearized perturbation analysis of the system has revealed that two types of oscillatory instabilities are possible; the first is due to a non-vanishing distortion at the interface, while the second is the result of a cyclic variation between viscous and thermal coupling. More recently, Colinet and Legros [22] using a weakly non-linear analysis have examined the oscillating convective structures that appear in the critical point of the Rayleigh–Bénard problem. Their analysis was complemented by numerical experiments which confirmed the appearance of a Hopf bifurcation.

Most studies relating to sidewall heating have been confined to the steady state. Ben Hadid and Roux reported [4–6] some results from their numerical simulation of oscillatory convection in low Prandtl number liquids in a shallow open cavity. Villers and Platten [8] presented their experimental and numerical studies of coupled buoyancy and Marangoni convection in acetone. Their experiments and numerical simulations both show the existence of three different states: monocellular steady states, multicellular steady states and a spatio-temporal structure. Mundrane and Zebib [23] recently provided a stability boundary in their defined *Re*–*Gr* plane for a single low-*Pr* number fluid layer. An energy analysis for the fluctuating part of the numerical 2-D-solution in order to discuss the instability mechanism was also presented. Doi and Koster [16] studied pure thermocapillary convection in two immiscible liquid layers with an upper free surface. Conditions for which motion in the lower encapsulated liquid layer would be suppressed were presented. Very recently, the present authors [11, 12] numerically simulated the steady and transient laminar combined buoyancy and thermocapillary convection in superposed immiscible liquid layers using the spline fractional step procedure (SMFS) [25]. In that study, the interface boundary conditions (maintaining the continuity of temperature, velocity, shear stress and heat flux at the interface) needed to be simultaneously satisfied. For medium Prandtl numbers and higher Rayleigh numbers, for example $Ra > 10^7$ or higher Marangoni numbers $Ma \geqslant 2\times10^4$, solutions with an initially quasi-periodic behaviour which persist for a long time were discovered.

The present investigation is devoted to a numerical simulation of oscillatory flows induced in two-layer systems of immiscible liquids with a free surface. The bottom layer is assumed to have the lower Prandtl number and particular attention is paid to the stabilizing effect of the upper encapsulate since this has an important bearing on the final crystalline structure of the lower layer. The development of the temperature and local flowfield has been documented. The simulation of the transient phase in a two fluid system reveals how the buoyancy and capillary forces (when in opposition) compete against each other, the final steady state depending on the dominancy of one over the other. The initiation and development of oscillatory flow and fundamental indications on how to reduce or even avoid these oscillations have been provided.

GOVERNING EQUATIONS

Thermocapillary convection which is induced by a combination of density differences in a gravitational field and by surface tension gradients, is governed by the two-dimensional Navier–Stokes equation and the energy equation for both fluids [$i = 1$ (below) and 2 (above)]. The non-dimensional equations in stream-function and vorticity form (using the Boussinesq approximation for the body forces) may be written:

$$\nabla_i^2 \Psi_i = -\Omega_i \tag{1}$$

$$\frac{\partial \Omega_i}{\partial t} + u_i \frac{\partial \Omega_i}{\partial x} + v_i \frac{\partial \Omega_i}{\partial y} = Pr_i \begin{Bmatrix} 1 \\ \bar{\alpha} \end{Bmatrix} \left[\nabla_i^2 \Omega_i + Ra_i \begin{Bmatrix} 1 \\ \bar{\alpha} \end{Bmatrix} \frac{\partial T_i}{\partial x} \right] \tag{2}$$

and

$$\frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial x} + v_i \frac{\partial T_i}{\partial y} = \begin{Bmatrix} 1 \\ \bar{\alpha} \end{Bmatrix} \nabla_i^2 T_i \tag{3}$$

with

$$\nabla_i^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{4}$$

and

$$u_i = \frac{\partial \Psi_i}{\partial y}, \quad v_i = -\frac{\partial \Psi_i}{\partial x} \tag{5}$$

where $\bar{\alpha} = \alpha_2/\alpha_1$ and $\bar{k} = k_2/k_1$.

Boundary conditions

For purposes of comparison between the results obtained in a single fluid layer and in a two fluid system, the present study considers two equal fluid layers each of height H'. The boundary conditions for the problem are:

for $x = 0$

$$u_i = v_i = \Psi_i = 0, \quad \Omega_i = -\frac{\partial^2 \Psi_i}{\partial x^2}, \quad \text{and} \quad T_i = -B/2$$

for $x = B$

$$u_i = v_i = \Psi_i = 0, \quad \Omega_i = -\frac{\partial^2 \Psi_i}{\partial x^2}, \quad \text{and} \quad T_i = B/2$$

for $y = 0$

$$u_1 = v_1 = \Psi_1 = 0, \quad \Omega_1 = -\frac{\partial^2 \Psi_1}{\partial y^2}, \quad \text{and} \quad \frac{\partial T_1}{\partial y} = 0 \tag{6}$$

and for $y = 2$

$$v_2 = \Psi_2 = 0, \quad \Omega_2 = \bar{\alpha} Ma_2 \frac{\partial T_2}{\partial x}, \quad \text{and} \quad \frac{\partial T_2}{\partial y} = 0. \tag{6'}$$

Neglecting any deflection of the interface between the two liquids, the boundary conditions maintaining the continuity of temperature and velocity at the interface ($y = 1$) are:

$$T_1 = T_2, \quad \frac{\partial T_1}{\partial y} = \bar{k} \frac{\partial T_2}{\partial y}$$

$$\Psi_1 = \Psi_2 \quad \text{and} \quad u_1 = u_2$$

$$\frac{\partial^2 \Psi_1}{\partial y^2} = \bar{\mu} \frac{\partial^2 \Psi_2}{\partial y^2} - Ma_1 \frac{\partial T}{\partial x} \tag{7}$$

or

$$\Omega_1 = \bar{\mu} \Omega_2 + Ma_1 \frac{\partial T}{\partial x} \tag{8}$$

where $\bar{\mu} = \mu_2/\mu_1$, Ma_1 is the interface Marangoni number and Ma_2 is the Marangoni number for the upper layer and

$$Ma_i = -\frac{\partial \sigma_i}{\partial T} \frac{(T'_h - T'_c) H'}{\mu_i \alpha_i} \tag{9}$$

σ_i being the surface tension.

The analysis above indicates that for immiscible two-fluid flow problems, the non-dimensional parameters of interest are Ra_1, Ra_2, Pr_1, Pr_2, $\bar{\alpha}$, $\bar{\mu}$, $\bar{k}$, Ma_1 and Ma_2.

NUMERICAL PROCEDURE

Since in two layer thermocapillary convection, each liquid layer influences the other through the interface conditions, the treatment of these conditions in the computational procedure is of extreme importance. In this respect, the spline fractional step procedure (SMFS) [25] is an improvement over existing methods. The essential advantage of the technique for the present problem lies in the fact that boundary conditions containing derivatives may be easily incorporated into the solution procedure since values of first or second derivatives may be evaluated directly and maintain the same degree of accuracy when the algorithm that represents the spline approximation to the governing equations (1)–(4) is constructed. The governing matrix system obtained is always tri-diagonal containing either function values or the first derivatives at the grid points. The SMFS schemes and the boundary conditions in discretized form may be obtained in direct fashion from the procedure detailed in earlier articles [10, 11, 26–28] and will therefore not be described further.

The time dependent non-linear coupled partial differential equations were solved by considering a 31×31, 41×41 or 81×41 grid depending on the different values of aspect ratio B. In order to accurately describe gradients that are expected to be steep in the boundary layer regions, a non-uniform grid in both the x- and y-directions was used. Accuracy of the solutions were verified by grid refinement. All computations were performed on a 486 based IBM compatible PC.

RESULTS AND DISCUSSION

Villers and Platten [15] introduced the ratio of certain physical quantities evaluated in the two layers:

$$Q_\alpha = [\rho\beta H'^2]_2/[\rho\beta H'^2]_1 \quad (10)$$

and

$$Q_\mu = [\mu/H']_2/[\mu/H']_1. \quad (11)$$

For the present case, if use is made of the above dimensionless parameters, equations (10) and (11) become:

$$Q_\alpha = \bar{\alpha}\bar{\mu} Ra_2/Ra_1$$

and

$$Q_\mu = \bar{\mu}$$

Q_α is related to the relative importance between the buoyancy forces in the upper and lower layers, and Q_μ is related to the viscous forces.

The present authors [11] introduced a new parameter

$$Ma^* = Ma_1 - 0.5\bar{\alpha}\bar{\mu}Ma_2 \quad (12)$$

which represents the combined effects of Marangoni forces acting at the interface between the two liquids and at the free surface. It was shown further that the new parameter is a unique thermocapillary quantity which influences the convection in the lower layer.

The interest in studying two fluid layers is due to the practical applications mentioned earlier, for example that of a liquid melt encapsulated by a protective molten material. This has generally resulted in a significant improvement in the quality of the final product. In the present study, we specifically consider the suppression of the undesired oscillations in the encapsulated low-Pr number fluid layer. This is dependent on the influence exerted by the buoyant convection in the upper layer over the encapsulated lower fluid layer (through coupling at the interface) and the combined effects of Marangoni forces (Ma^*) acting at the interface between the two liquids and at the free surface.

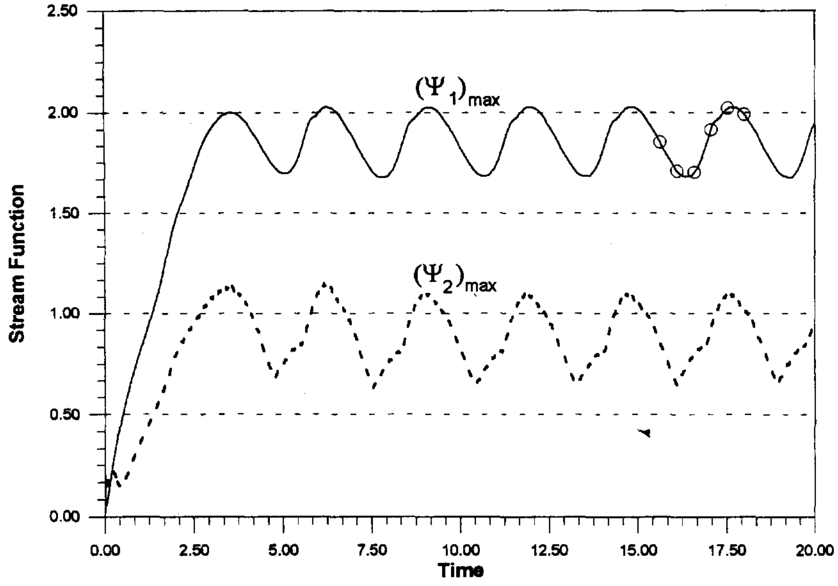
Many combinations of liquids are possible to provide an immiscible liquid system. Combined buoyancy and thermocapillary convection in superposed immiscible liquid layers using some selected fluids have been investigated experimentally [7, 16, 17, 24]. In the present study, with no loss of generality, we used $Pr = 0.015$ for the encapsulated liquid of low Prandtl number, and $Pr = 1$ for the liquid encapsulant.

The present combination of two superposed immiscible fluid layers, may result in oscillatory regimes as detailed below. Figure 1(a) presents the time-variation of the maximum stream function in both layers for $Ra_1 = 600$, $Ma_1 = Ma_2 = 0$, $Q_\alpha = 0.01$, $Q_\mu = 0.1$, $Pr_1 = 0.015$, $Pr_2 = 1$ and $B = 4$ in a rectangular cavity with differentially heated end walls ($T_c = -B/2$ and $T_h = B/2$). The non-linear character of the oscillation is clear.

Figure 1(b) provides the time variation of the total mean Nusselt number $\overline{Nu}$ at the two endwalls and at the midplane, respectively. It is noted that the variation of the total mean Nusselt number $\overline{Nu}$ is stronger near the cold wall than at the midplane, while near the hot wall this variation is comparatively weak. Figure 2 illustrates the variation of the isotherms and streamlines in the lower layer at six instants [the small circles shown in Fig. 1(a)] over a period of oscillation.

It is not surprising that the variation of the flow patterns in the lower layer are similar to those obtained in a single fluid layer by Ben Hadid and Roux [6]. This is a particular case where the buoyancy force and the viscous force in the upper layer are very small compared with the lower one. In other words, the influence of the upper layer on the lower one through the interface is very weak so that oscillations in the lower layer cannot be avoided.

As was expected, increasing the value of Q_μ (relating viscous forces) or Q_α (relating buoyancy forces), i.e. increasing the influence of the upper layer on the lower layer, may be beneficial in enhancing stability. In fact, for pure gravitational convection, the flow behaviour in the lower layer is influenced by the viscous effects and the heat transfer through the interface between the two layers. The higher the value of Q_μ, the stronger the stabilizing influence on the lower layer. As an example the following values: $Ra_1 = 600$, $Pr_1 = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 1$ with $B = 4$, while $Q_\alpha = Q_\mu = 1$, i.e. with buoyancy and viscous forces of the same order in both layers, the steady state flow obtained will be as shown in Fig. 3(a)–(b). These figures present the time history of the maximum stream function in both layers $(\Psi_1)_{max}$ and $(\Psi_2)_{max}$ and the total mean Nusselt number $\overline{Nu}$ respectively. $(\Psi_1)_{max}$ in the upper layer quickly approaches its maximum value at about a dimensionless time of $t = 8$, $(\Psi_2)_{max}$ in the lower layer approaches steady state, and the difference between the mean Nusselt number at the sidewalls and at the midplane, i.e. $\overline{Nu}_w$ and $\overline{Nu}_0$ is less than 1%. At the initial stage, the total mean Nusselt number at the sidewall immediately attains its maximum value, then rapidly diminishes and at a time of about $t = 1.08$, the total mean Nusselt number at the hot wall approaches a minimum and then again gradually increases towards its steady state. However, the total mean Nusselt number at the midplane departs from zero only at $t = 0.12$, then rapidly attains its maximum and finally tends towards its



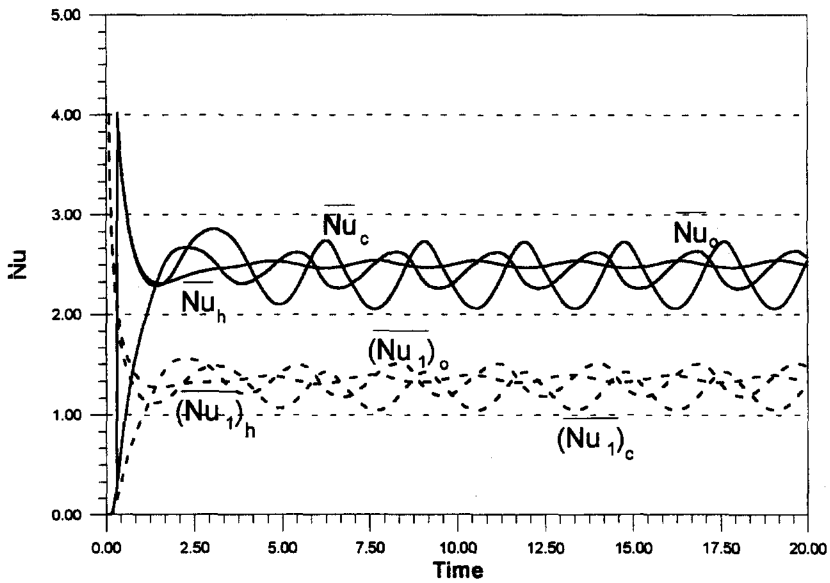


Fig. 1. $Ra_1 = 600$, $Q_\alpha = 0.015$, $Q_\mu = 0.1$, $Pr_1 = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 1$ and $B = 4$: (a) time variation of the maximum stream function; (b) time variation of Nusselt numbers at the two endwalls (Nu_h and Nu_c) and at the midplane (Nu_0) for two layers.

steady state value with very small oscillations. This is similar to the features observed in cavities filled with a single fluid at medium Prandtl numbers.

Figure 4(a) provides the time history of the maximum stream function for $Ra_1 = 600$, $Pr_1 = 0.015$, $Pr_2 = 1$ and $B = 4$ with (a) $Q_\alpha = 0.1$, $Q_\mu = 10$ and $\bar{\alpha} = 0.1$; and (b) $Q_\alpha = 10$, $Q_\mu = 0.1$ and $\bar{\alpha} = 1$ respectively. Comparing these results with those for the case where $Q_\alpha = 0.01$ and $Q_\mu = 0.1$ (shown in Fig. 1a), the stabilizing effects may be noted. The relatively higher viscous forces in the upper layer ($Q_\mu = 10$) result in a steady state behaviour in the lower layer while the relatively higher buoyancy forces in the upper layer ($Q_\alpha = 10$), results in a weakening of the oscillation in the lower layer.

Figure 4(b) indicates the time history of the horizontal velocity at the centre point of the interface for $Ra_1 = 600$, $Pr_1 = 0.015$, $Pr_2 = 1$ and $B = 4$ with

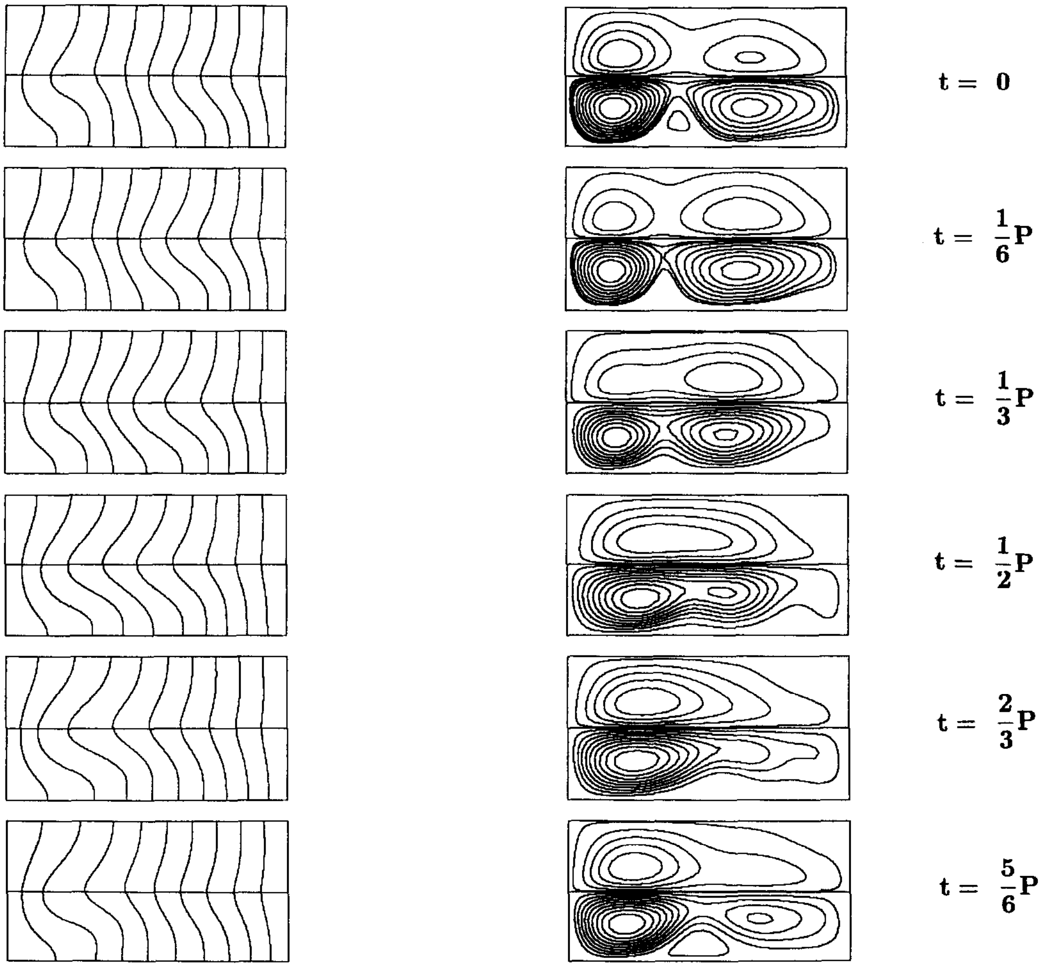


Fig. 2. Time variation of the isotherms and streamlines at six instants over a period of oscillation in Fig. 1(a).

$Q_\mu = 10$ and $Q_\alpha = 0.1$, 1 and 10, respectively. With an increase in the value of Q_α, the velocity direction is changed; for higher values of Q_α, the direction is dominated by the convection in the upper layer. The steady state isotherms and streamlines are presented in Fig. 5.

As for the case of the single fluid layer, addition of Marangoni forces to the buoyant convection may also be beneficial to the stability. However, it is important to point out that in a two fluid layer system, the combined Marangoni number Ma^* defined in equation (12) plays a dominant role in the stabilizing effects. It is similar to the influence of Marangoni forces at the free surface for a single fluid layer.

Figure 6(a)–(b) present the steady state isotherms and streamlines for $Ra_1 = 600$, and $Ma_2 = 0$, $Q_\alpha = 0.01$, $Q_\mu = 0.1$, $Pr_1 = 0.015$, $Pr_2 = 1$ with $B = 4$, while $Ma^* = -34(Re = -2.4 \times 10^3)$ and $Ma^* = 30(Re = 2.0 \times 10^3)$.

It is also noted that the flow patterns in the lower layer with $Ra = 600$ and $Re = -2.4 \times 10^3$ and $Re = 2 \times 10^3$ are similar to the results obtained in a single fluid layer by Ben Hadid and Roux [6].

For $Ma^* = -34$, the fluid at the interface tends to flow from the cold to the hot wall, i.e. in the lower layer opposite in direction to that generated by the buoyancy force. As a result, two counter-rotating vortices are generated by the thermocapillary flow close to the interface. While $Ma = 30$, the fluid at the interface also tends to flow from the hot to the cold wall reinforcing the gravitational convection. The flow is thus further accelerated at the interface and additional energy is available to maintain the two principal cells in the lower layer.

The addition of Marangoni forces at the free surface (represented by the Marangoni number Ma_2) influence the lower layer by coupling with the flow in the upper layer through viscous stress across the interface. This depends strongly on Q_μ. In the present study, using the above parameter with $Ma_2 = 400$ or $Ma_2 = -400$ while $Ma^* = 0$, results in oscillatory flow being encountered.

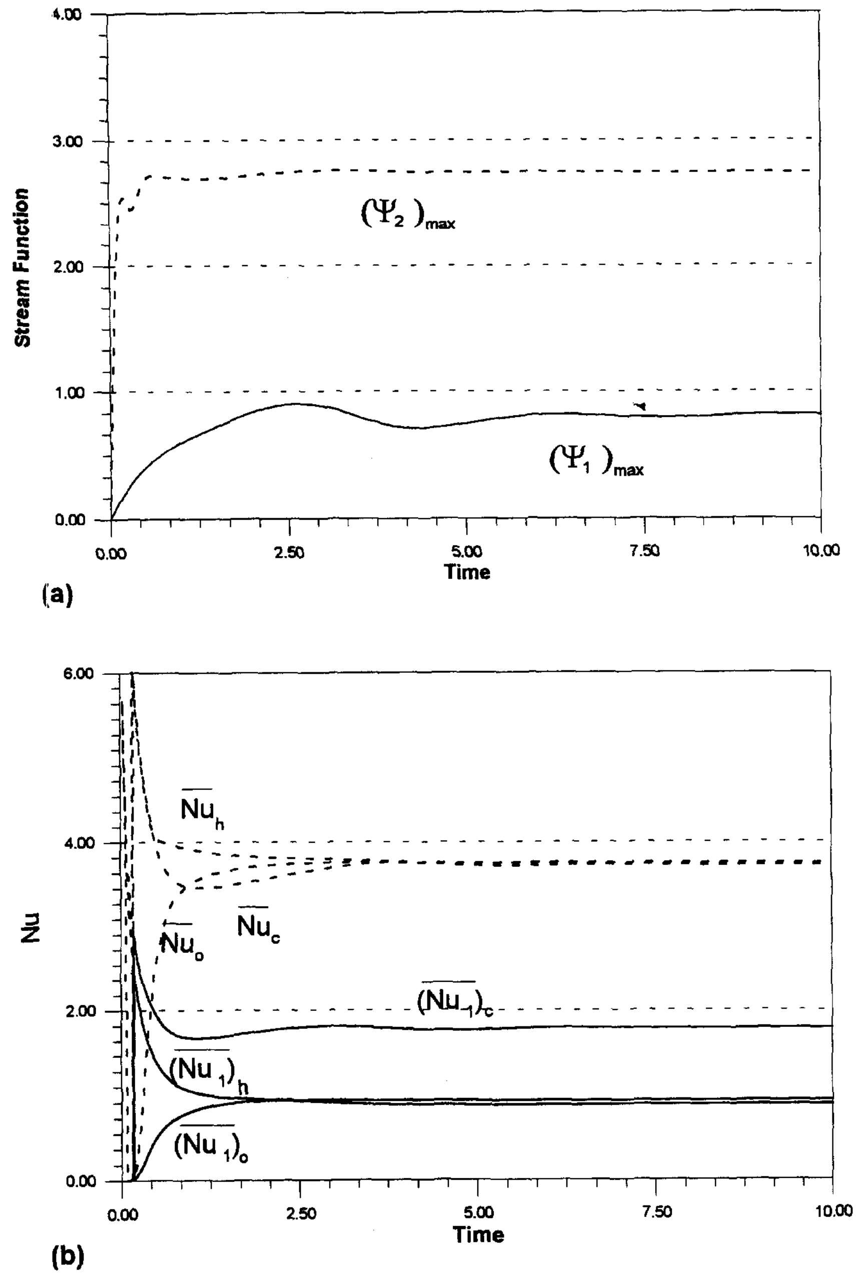
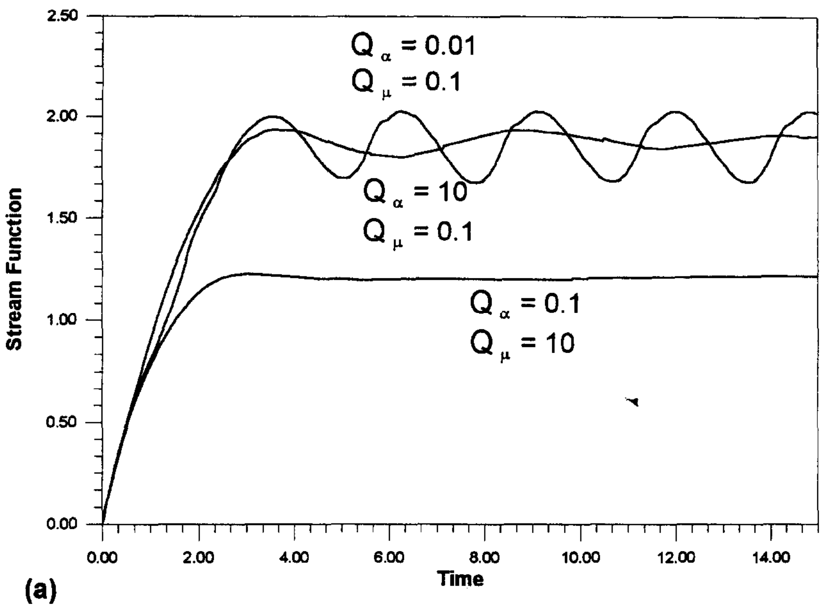


Fig. 3. $Ra_1 = 600$, $Q_\alpha = 0.015$, $Q_\mu = 0.1$, $Pr_1 = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 1$ and $B = 4$: (a) time variation of the maximum stream function; (b) time variation of Nusselt numbers at the two endwalls (Nu_h and Nu_c) and at the midplane (Nu_0) for two layers.

Figure 7(a) presents the steady state flow patterns in the absence of buoyant forces, i.e. theoretically zero gravity for $Ma^* = 0$ and $Ma_2 = 100$. $Ma^* = 0$ corresponds to a special case, theoretically no fluid motion will arise in the lower layer for an infinite twin layer system, and so any heat transfer has to occur by pure conduction, the resultant temperature expression being $T = Cx$ along the interface [11]. However, due to sidewall effects, in a finite cavity, the difference between the numerical results and the analytical solution may often be discerned visually, especially for low-Prandtl number fluids.

Figure 7(b, c) show the steady state flow patterns for $Ma^* = 0$ and $Ma_2 = 100$, $Ma^* = 0$ and $Ma_2 = -100$, with $Ra_1 = 600$, $Q_\alpha = 1$, $Q_\mu = 10$, $Pr_1 = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 0.1$ and $B = 4$. In the two cases, the flow in the lower layer is dominated by the buoyant force and the viscous effects at the interface,



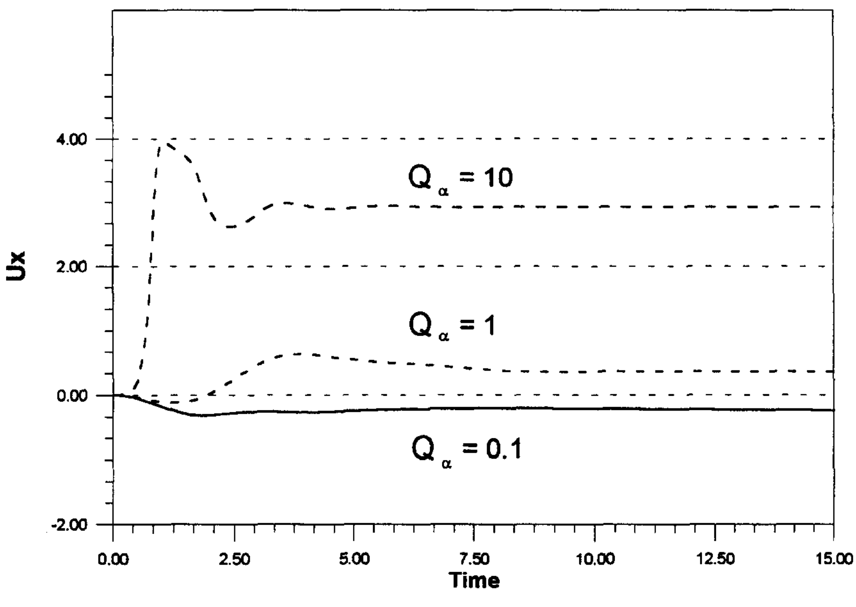


Fig. 4. $Ra_1 = 600$, $Pr_1 = 0.015$, $Pr_2 = 1$, and $B = 4$: (a) time history of the maximum stream function for $Q_\alpha = 0.1$ and $Q_\mu = 10$, $Q_\alpha = 10$ and $Q_\mu = 0.1$, and $Q_\alpha = 0.01$ and $Q_\mu = 0.1$; (b) time history of the horizontal velocity at the center point of the interface for $Q_\mu = 10$ and $Q_\alpha = 0.1$, 1 and 10.

the two principal cells in the lower layer being maintained.

CONCLUSIONS

Numerical simulation of oscillatory flows induced in a two-layer system of immiscible liquids with a free surface (the liquid in the lower layer being of lower Prandtl number), has been studied. The numerical results indicate that oscillatory flow in a single layer of low Prandtl number fluid may transform into a stationary steady-state behaviour after encapsulation with a fluid of higher Prandtl number even in the absence of Marangoni forces, except when the buoyancy and the viscous forces in the upper layer are very small compared with those of the lower one (a particular case being a gas). The numerical experiments also demonstrate that the addition of combined Marangoni forces to the gravitational convection may be beneficial in enhancing the stability.

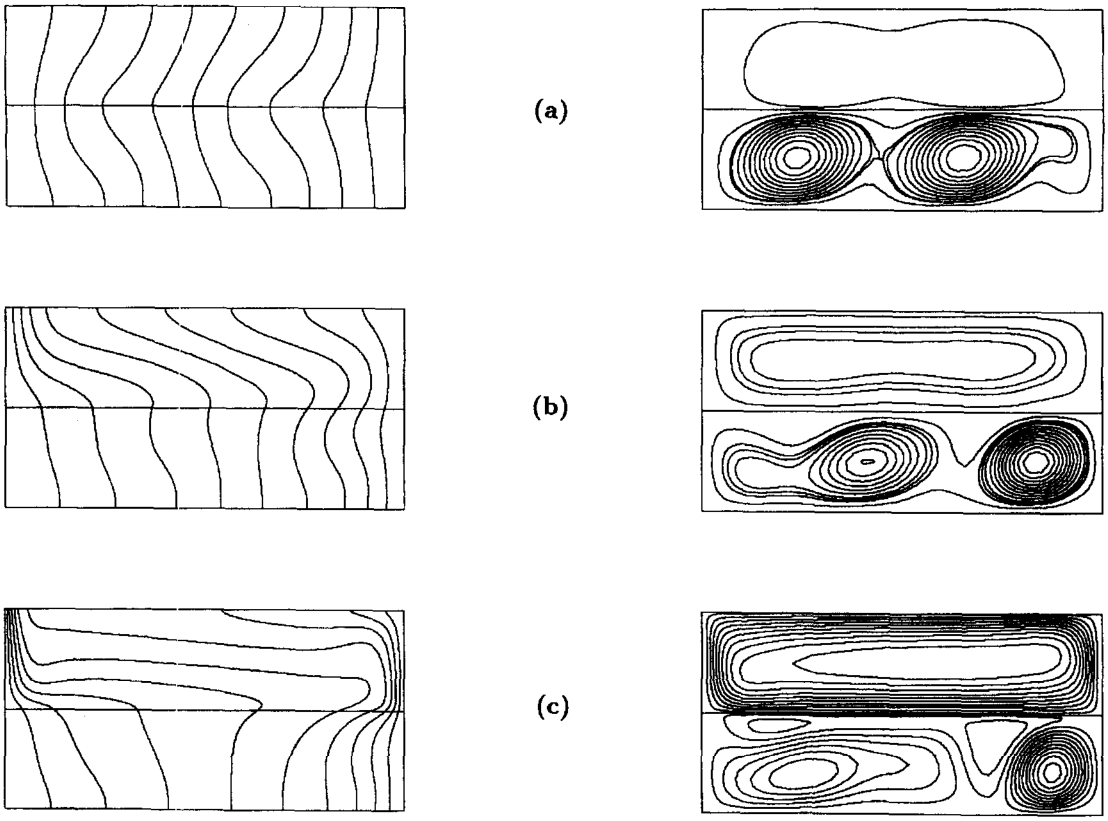


Fig. 5. Steady state isotherms and streamlines [same parameters as Fig. 4(b)].

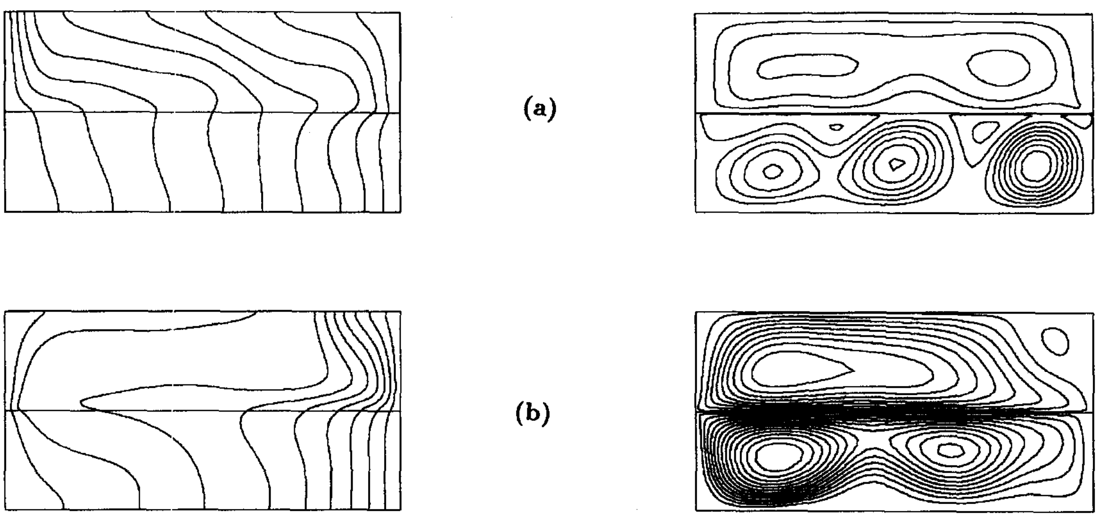


Fig. 6. Steady state isotherms and streamlines for $Ra_1 = 600$, and $Ma_2 = 0$, $Q_\alpha = 0.01$, $Q_\mu = 0.1$, $Pr_1 = 0.015$, $Pr_2 = 1$ with $B = 4$: (a) $Ma^* = -34(Re = -2.4 \times 10^3)$; (b) $Ma^* = 30(Re = 2.0 \times 10^3)$.

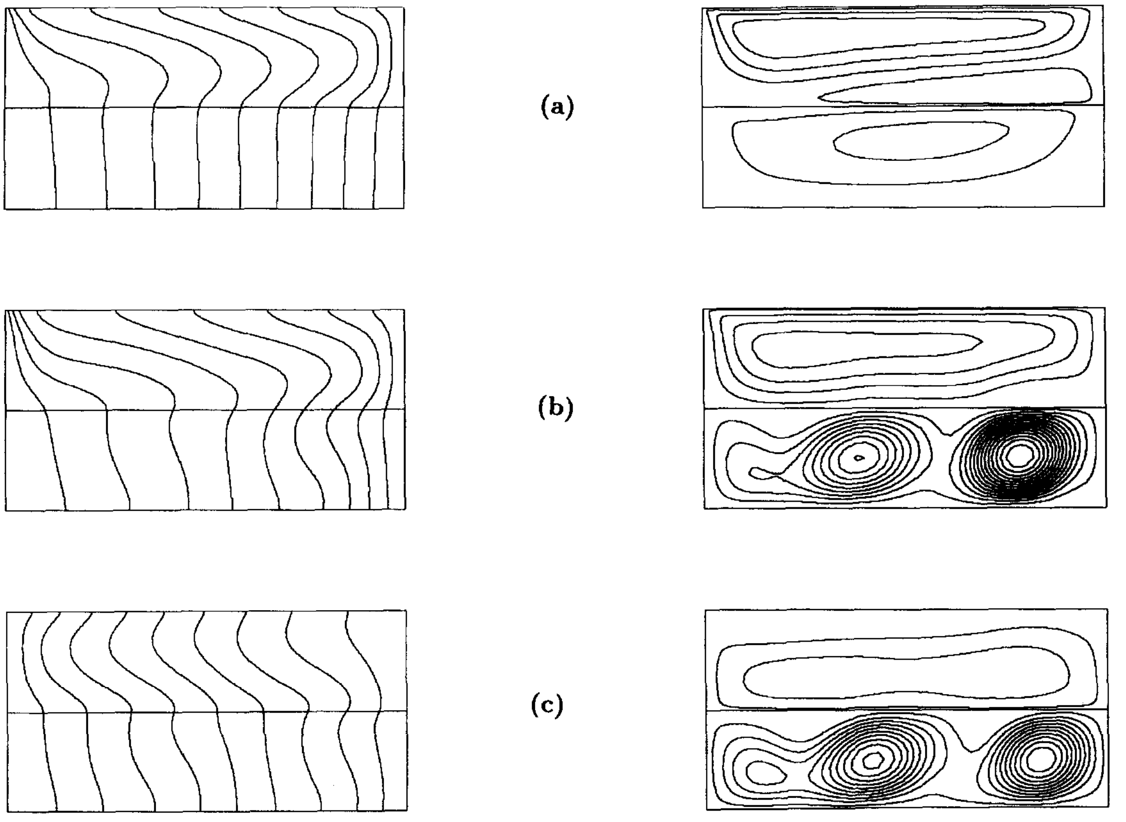


Fig. 7. Steady state isotherms and streamlines: (a) $Ra = Ma^* = 0$, $Ma_2 = 100$ and $\bar{\alpha} = 0.1$; (b) $Ma_2 = 100$ with $Ma^* = 0$, $Ra_1 = 600$, $Q_\alpha = 1$, $Q_\mu = 10$, $Pr_{21} = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 0.1$ and $B = 4$; (c) $Ma_2 = -100$ with $Ma^* = 0$, $Ra_1 = 600$, $Q_\alpha = 1$, $Q_\mu = 10$, $Pr_1 = 0.015$, $Pr_2 = 1$, $\bar{\alpha} = 0.1$ and $B = 4$.

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